Closing Today: HW_5A,5B (7.1,7.2)
Closing Fri: HW_5C (7.3)
Office Hours Today: 1:30-3:00pm (Smith 309)
Math Study Center: 9:30am – 9:30pm (Com B-014)
CLUE Tutors: 7:00pm – midnight (Mary Gates Hall)

Entry Task: Integrate

$$\int \sqrt{9+x^2}\,dx$$

7.3 Trigonometric Substitution Summary

CASE	SUBSTITUTION
$\sqrt{a^2 - x^2}$	x = asin(θ), dx = acos(θ)dθ $\sqrt{a^2 - a^2\sin^2(\theta)} = \text{a}\cos(\theta)$
$\sqrt{a^2 + x^2}$	x = atan(θ), dx = a $\sec^2$(θ)dθ $\sqrt{a^2 + a^2\tan^2(\theta)} = \text{a}\sec(\theta)$
$\sqrt{x^2 - a^2}$	x = asec(θ), dx = asec(θ)tan(θ)dθ $\sqrt{a^2\sec^2(\theta) - a^2} = \text{a}\tan(\theta)$

Don't forget you can quote these:

$$\int \tan(x)\,dx = \ln|\sec(x)| + C$$

$$\int \sec(x)\,dx = \ln|\sec(x) + \tan(x)| + C$$

$$\int \sec^3(x)\,dx =$$

$$\frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(\mathrm{x}) + \tan(\mathrm{x})| + C$$

Example: (Integrating under a half-circle)
Evaluate

$$\int \sqrt{4 - x^2}\,dx$$

$$\int_0^2 \sqrt{4 - x^2}\,dx$$

If you encounter a `middle term'

$$\sqrt{ax^2 + \boldsymbol{bx} + c}.$$

Complete the square:

1. $x^2 + 10x =$

2. $2x^2 - 12x + 22 =$

3. $14 - 8x - x^2 =$

Example: Evaluate

$$\int \frac{x}{\sqrt{34 - 6x + x^2}}\, dx$$

7.4 Partial Fractions

Goal: Learn a general method to integrate rational functions (a polynomial over a polynomial). This is an important algebraic method for simplifying fractions which you will use in many, many other math courses.

Motivation:

$$\int \frac{x^3 + 4x - 4}{x^2(x^2+4)}\,dx \quad = ??$$

We will learn to break up the fraction into:

$$\frac{x^3 + 4x - 4}{x^2(x^2+4)} = \frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^2+4}$$

Then we integrate each part separately.

Here are examples of the **<u>only integrals</u>** you need for this section:

$$\int \frac{1}{2x+5}\,dx \quad = \frac{1}{2}\ln|2x+5| + C$$

$$\int \frac{1}{(x-4)^2}\,dx \quad = -\frac{1}{x-4} + C$$

$$\int \frac{1}{(x+7)^3}\,dx \quad = -\frac{1}{2}\frac{1}{(x+7)^2} + C$$

$$\int \frac{1}{x^2+9}\,dx \quad = \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$$

$$\int \frac{x}{x^2+9}\,dx \quad = \frac{1}{2}\ln|x^2+9| + C$$

The method will reduce **every** rational function problem to one of these.

Partial Fraction Decomposition

Step 1: Is the fraction in lowest terms?

The highest power on top needs to be smaller than the highest power on bottom. If not, divide.

Example:

$$\int \frac{x^2 + x}{x + 3}\,dx$$

<u>Partial Fractions Method Summary</u>

Step 1: Divide, if needed (on previous page).

Step 2: Factor Denominator and write out form of decomposition.

i) Distinct Linear:

$$\frac{x^2-3}{x(x-1)(x+4)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+4}$$

ii) Repeated Linear:

$$\frac{5+2x}{(x+3)(x-2)^3} = \frac{A}{x+3} + \frac{B}{x-2} + \frac{C}{(x-2)^2} + \frac{D}{(x-2)^3}$$

iii) Irreducible Quadratic:

$$\frac{4x}{(x+1)(x^2+9)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+9}$$

Step 3: Solve for A, B, C ….

Step 4: Integrate

Example:

$$\int \frac{x+1}{x^2-4}\,dx$$

Example:

$$\int \frac{x+1}{x^3+3x^2}\,dx$$

Example:

$$\int \frac{x^2 - x + 6}{x^3 + 3x}\,dx$$

Example:

$$\int \frac{x}{x^2 + 4x + 5}\,dx$$